

Distributed Joint Source-Channel Coding for arbitrary memoryless correlated sources and Source coding for Markov correlated sources using LDPC codes

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Abstract—In this paper, we give a distributed joint source channel coding scheme for arbitrary correlated sources for arbitrary point in the Slepian-Wolf rate region, and arbitrary link capacities using LDPC codes. We consider the Slepian-Wolf setting of two sources and one destination, with one of the sources derived from the other source by some correlation model known at the decoder. Distributed encoding and separate decoding is used for the two sources. We also give a distributed source coding scheme when the source correlation has memory to achieve any point in the Slepian-Wolf rate achievable region. In this setting, we perform separate encoding but joint decoding.

Index Terms—Distributed Source coding, joint source-channel coding, LDPC codes, Slepian-Wolf.

I. INTRODUCTION

DISTRIBUTED source coding (DSC) refers to the compression of the outputs of two or more physically separated sources that do not communicate with each other (hence the term distributed coding). These sources send their compressed outputs to a central point (e.g., the base station) for joint decoding. Driven by a host of emerging applications (e.g., sensor networks and wireless video), DSC has recently become a very active research area - more than 30 years after Slepian and Wolf laid its theoretical foundation [1].

Wyner first realized the close connection of DSC to channel coding and suggested the use of linear channel codes as a constructive approach for Slepian-Wolf coding in his 1974 paper [25]. Wyner's scheme was only recently used in [2] for practical Slepian-Wolf code designs based on conventional channel codes like block and trellis codes. If the correlation between the two sources can be modelled by a binary channel, Wyner's syndrome concept can be extended to all binary linear codes; and state-of-the-art near-capacity channel codes such as turbo [26] and LDPC codes [23] [24] can be employed to approach the Slepian-Wolf limit.

Slepian-Wolf rate region is the rate region in which there exist encoding and decoding schemes such that reliable transmission can take place, which was given in [1] as $R_1 \geq H(X|Y)$, $R_2 \geq H(Y|X)$, and $R_1 + R_2 \geq H(X, Y)$, where R_1 and R_2 are the source encoding rates of the two sources X and Y , and $H(X|Y)$ and $H(Y|X)$ the conditional entropies of the sources, and $H(X, Y)$ the joint entropy of the two sources. The most important points in the Slepian-Wolf rate region are

the asymmetric point (in which one source, say X , is encoded at the rate that equals its unconditional entropy ($H(X)$) and the other source, say Y , is encoded at the rate equal to its conditional entropy ($H(Y|X)$)), and the symmetric point (in which both sources are encoded at the same rate $H(X, Y)/2$). Constructive approaches (e.g., [5]) have been proposed for the symmetric point of the boundary of the Slepian-Wolf rate region, for the asymmetric point of the boundary of the Slepian-Wolf rate region (example [2][7][12] [18]) and also for any point between on the boundary (for example [4][13]). Most of the cases in the literature deal with correlation model being a binary symmetric channel, though some work on arbitrary correlation has also been done (for example [4]).

After the practical scheme of [2], there was a lot of work in the direction of the practical code construction due to its application in sensor networks [5]. Some of the works relating to this field are [4][6][7][8][9][10][11][12]. In [4], practical construction using syndrome approach was given for arbitrarily correlated sources. There is also another scheme not using the syndrome approach, but rather encoding in the same way as the channel coding, by multiplying at the encoder by the generator matrix as is given in [13] (The idea of using such approach is also there in [3]). They consider in the paper source correlation model as BSC with error probability p . To encode, a non-uniform systematic LDPC code with generator matrix of size $n \times n(1+h(p))$ is used (where $h(p)$ is the binary entropy of p), and at the encoder of the first source, say X_1 , $1/a$ fraction of the information bits, and $(1-1/a)$ fraction of the parity bits are sent. At the encoder of the other source, say X_2 , the remaining $(1-1/a)$ fraction of information bits and the remaining $1/a$ fraction of the parity bits are sent. Decoding is performed as follows: At decoder of X_1 , we have $1/a$ fraction of information bits of X with infinite likelihood, $(1-1/a)$ fraction of parity bits with infinite likelihood, $(1-1/a)$ fraction of information bits with $\ln((1-p)/p)$ likelihood, from which we decode X_1 , and similarly we can decode X_2 . Note that the remaining parity bits from X_2 are not used in the decoding of X_1 , they are considered as being punctured. They also suggest to use unequal error protection for LDPC since the information bits are more important. More important bits have higher degrees while the rest have lower degree. Hence, they design systematic LDPC code with

distinct variable node degree distributions for information and parity bits. Our scheme will extend upon this basic idea of doing source coding in the same way as traditional channel coding scheme.

There has also been some work in the direction of joint source channel decoding of correlated sources over noisy channels (for example [6], [19], [22], [20], and [21]) but none of the work till now covers the problem in that generality using LDPC codes. It has been proven in [11] that LDPC codes can approach the Slepian Wolf bound for general binary sources, and hence the choice of LDPC codes [23][24] in this work.

Most of the cases in the literature deal with correlation model being a binary symmetric channel, though some work on arbitrary correlation has also been done (for example [4]) and also for correlation having Markov structure (for example [16] [18]). To the best of our knowledge, no work for arbitrary point in the Slepian-Wolf rate region is there in the literature although there are attempts for asymmetric point in the rate region(for example [16] [18]).

The paper is organized as follows: We will start with giving the scheme of joint source channel coding for the source correlation model being arbitrary memoryless with arbitrary link capacities at arbitrary point in the Slepian-Wolf Rate region in Section II. Then, we give some simulation results for this scheme in Section III. Following this, we give a scheme for doing distributed source coding at arbitrary rates in the Slepian-Wolf Rate Achievable region when the source correlation has memory in Section IV and some simulations for the scheme are given in Section V. Finally, we give some concluding remarks in Section VI.

II. JOINT SOURCE CHANNEL CODING SCHEME FOR ARBITRARY SOURCE CORRELATION

Consider first that we encode X_1 as follows: X_1 is fed into systematic LDPC code of Rate R_1 . The encoder sends the corresponding parity bits and a $1/a$ fraction of the information bits. These parity bits involve the bits needed for source as well as channel. Let P_1 be the parity bits needed for source and P_3 the parity bits needed for the channel for source X_1 . The compression rate of source X_1 is $R_{X_1} = \frac{k/a+P_1}{k}$, and the corresponding LDPC code has rate $R_1 = \frac{k}{k+P_1+P_3}$ where k is the number of bits of X_1 and of X_2 at the input. The same procedure with a few modifications is applied at the source X_2 . For this source, we use a systematic LDPC code of rate R_2 . The encoder sends the related parity bits and the remaining $1-1/a$ fraction of the information bits. Similarly, assume that the parity bits are P_2 for the source coding and P_4 for the channel coding at the source X_2 , we have the compression rate and the LDPC code rate equal to $R_{X_2} = \frac{(a-1)k/a+P_2}{k}$ and $R_2 = \frac{k}{k+P_2+P_4}$ respectively.

The procedure as we discussed above involves two LDPC codes. However, we are interested in designing a single channel code for both sources. So, we will make a single LDPC matrix having the information bits, and all parity bits corresponding to P_1, P_2, P_3 , and P_4 .

Lemma 2.1: $P_1 + P_2 = k(R_{X_1} + R_{X_2} - 1)$

Proof: Add R_{X_1} and R_{X_2} given above to get the required

Lemma 2.2: $P_3 + P_4 = k[R_{X_1}(\frac{1}{R_{c_1}} - 1) + R_{X_2}(\frac{1}{R_{c_2}} - 1)]$ ■

Proof:

$$\begin{aligned} R_{c_1} &= \frac{kR_{X_1}}{kR_{X_1} + P_3} \\ R_{c_2} &= \frac{kR_{X_2}}{kR_{X_2} + P_4} \\ \Rightarrow P_3 &= kR_{X_1}(\frac{1}{R_{c_1}} - 1), P_4 = kR_{X_2}(\frac{1}{R_{c_2}} - 1) \end{aligned}$$

Add P_3 and P_4 to get the required result. ■

Let the rate of the LDPC code needed is R . With this parity check matrix, we will generate required parity bits for each source. Now, as we can see, the modified algorithm is to encode both the sources with a single LDPC code, and at encoder of X_1 , send the first fraction $1/a$ of the information bits, and the first $1/b$ fraction of the parity bits, and at encoder of X_2 , send the remaining $1-1/a$ information bits and last $1/c$ fraction of the parity bits (Figure 1). Also keep in mind that to satisfy the Slepian-Wolf Compression bounds, R_{X_1} and R_{X_2} (the source compression rates of the two sources) have to be in the Slepian-Wolf rate region. At the decoder of the each source, we use the information bits coming from both the sources, and the parity bits from that source, and the remaining bits are considered as punctured. The parameters $a, b, c, R, R_{c_1}, R_{c_2}$ (the channel rates of the two channels), R_{X_1} and R_{X_2} are related as in (1). C_f and C_b are the capacities of the channel from the source X_1 to X_2 and from source X_2 to source X_1 respectively.

$$\begin{aligned} (1 - \frac{1}{c})(\frac{1}{R} - 1) &= (1 - \frac{R_{c_1}}{R_{c_2}}C_f)(1 - \frac{1}{a}) \\ (1 - \frac{1}{b})(\frac{1}{R} - 1) &= \frac{1}{a}(1 - \frac{R_{c_2}}{R_{c_1}}C_b) \\ R_{X_1} &= R_{c_1}[\frac{1}{a} + \frac{\frac{1}{R} - 1}{b}] \\ R_{X_2} &= R_{c_2}[1 - \frac{1}{a} + \frac{\frac{1}{R} - 1}{c}] \end{aligned} \quad (1)$$

As we can see that punctured bits can be further decreased by taking the total parity bits as the maximum number of parity bits being used and which equals maximum of $P_1 + P_3$ and $P_2 + P_4$. Hence we perform the second step according to (2).

$$\begin{aligned} R' &\leftarrow \frac{R}{R + \max(\frac{1-R}{b}, \frac{1-R}{c})} \\ b &\leftarrow b \times \frac{R(1-R')}{R'(1-R)} \\ c &\leftarrow c \times \frac{R(1-R')}{R'(1-R)} \\ R &\leftarrow R' \end{aligned} \quad (2)$$

The encoding and decoding functions are explained in detail below:

Encoding:

The scheme is explained in the figure 1. At the encoder of X_1 , we send the first $1/a$ fraction of information bits and first

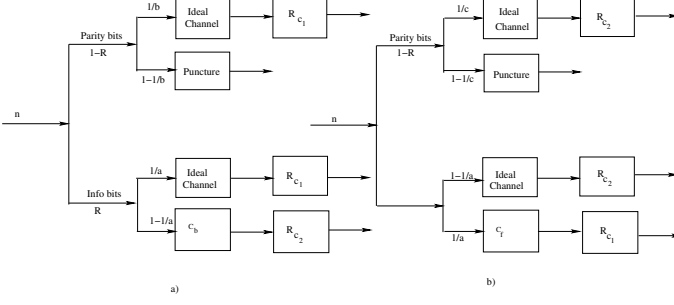


Fig. 1. Parallel Channel for (a) source X_1 and (b) source X_2

$1/b$ fraction of the parity bits. At the encoder of source X_2 , we send the remaining $1-1/a$ fraction of information bits, and last $1/c$ fraction of parity bits. The LDPC matrix made is of rate R . We have four variables here a, b, c, R for given R_{X_1} , R_{X_2} , R_{c_1} and R_{c_2} for which we use the set of equations (1) to solve them explicitly. It can also be seen that

$$R_1 = \frac{k}{k + k(\frac{1}{R} - 1)(\frac{1}{b})} = \frac{1}{1 + (\frac{1}{R} - 1)(\frac{1}{b})}$$

$$R_2 = \frac{k}{k + k(\frac{1}{R} - 1)(\frac{1}{c})} = \frac{1}{1 + (\frac{1}{R} - 1)(\frac{1}{c})}$$

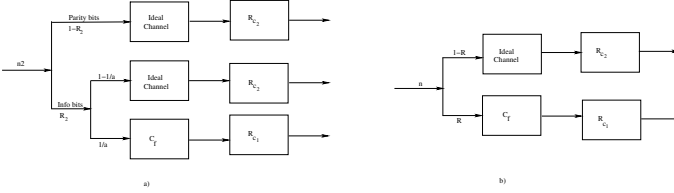


Fig. 2. (a) Equivalent Model of Figure 1(b), and (b) Special case of this model as $c=1$

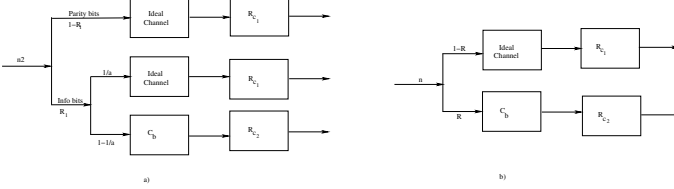


Fig. 3. (a) Equivalent Model of Figure 1(a), and (b) Special case of this model as $b=1$

Consider channels of Figure 1(b). In Figure 2(a), we have an equivalent model with $n_2 = n - (1 - 1/c)(n - k)$, where $k = nR$, and the parity bits are P_2 and P_4 . According to [14], the performance of decoder do not change after puncturing if R/C do not change. It is easy to see using (1) that the model of source X_2 is similar to test model in Figure 2(b) when $c = 1$. We will show that the decoder performance of the model of Figure 1(b) and figure 2(b) are the same in the next few lemmas. Similarly, we will show that the decoder performance of the model of Figure 1(a) and figure 3(b) (its equivalent model when $b=1$) which means that decoder performance do not change with the choices of a, b, c .

Lemma 2.3: The ratio of rate and capacity are same for Figure 2(a) (or Figure 1(b)) and Figure 2(b) are the same

Proof: As

$$C_2 = (1 - R_2)R_{c_2} + R_2R_{c_2}(1 - \frac{1}{a}) + \frac{R_2}{a}C_fR_{c_1}$$

$$R_2 = \frac{R}{R + (1 - R)(\frac{1}{c})}$$

$$C = (1 - R)R_{c_2} + RC_fR_{c_1}$$

After some manipulations, we get $\frac{R_2}{C_2} = \frac{R}{C}$ using Equations (1). ■

Lemma 2.4: The ratio of rate and capacity are same for Figure 1(a) and Figure 3(b) are the same.

Proof: As

$$C_1 = (1 - R_1)R_{c_1} + R_1(1 - \frac{1}{a})C_bR_{c_2} + R_{c_1}\frac{R_1}{a}$$

$$R_1 = \frac{R}{R + (1 - R)(\frac{1}{b})}$$

$$C = (1 - R)R_{c_1} + RC_bR_{c_2}$$

After some manipulations, we get $\frac{R_1}{C_1} = \frac{R}{C}$ using Equations (1). ■

It is also easy to see that the bits sent satisfy equation (1), the total bits that are sent through encoder X_1 are

$$\frac{k}{a} + k(\frac{1}{R} - 1)(\frac{1}{b}) = k\frac{R_{X_1}}{R_{c_1}}$$

and the total bits that are sent through encoder X_2 are

$$k(1 - \frac{1}{a}) + k(\frac{1}{R} - 1)(\frac{1}{c}) = k\frac{R_{X_2}}{R_{c_2}}$$

Hence all the equations in equation (1) makes good sense. As we can see that punctured bits can be further decreased by taking the total parity bits as the maximum number of parity bits being used and which equals maximum of $P_1 + P_3$ and $P_2 + P_4$. Hence we perform the second step according to equation (2).

We still need to find C_f and C_b for solving the above equations. Consider the forward model as in figure 4. The

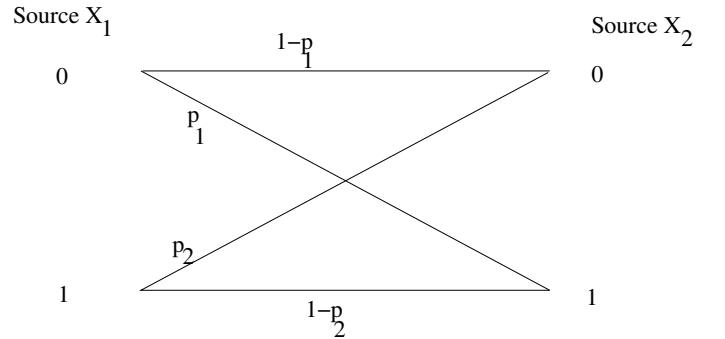


Fig. 4. Forward Model

capacity of the forward model is given by

$$C_f = h\left(\frac{1}{1 + 2^{\frac{h(p_1) - h(p_2)}{1 - p_1 - p_2}}}\right) - \beta h(p_1) - (1 - \beta)h(p_2)$$

where $\beta = \frac{\frac{1}{1+2} \frac{h(p_1) - h(p_2)}{1-p_1-p_2} - p_2}{1-p_1-p_2}$
 If the probability that $(X_1 = 0)$ is α , then the backward channel is given as in figure 5

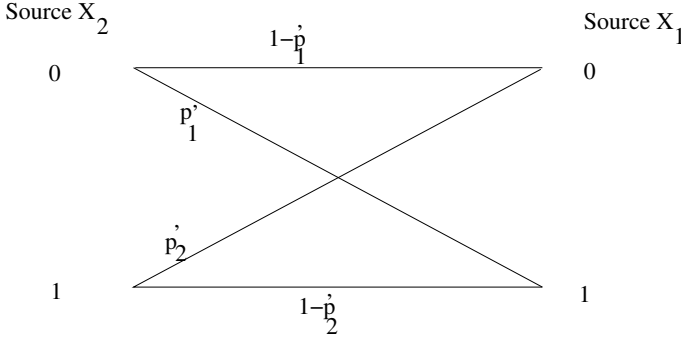


Fig. 5. Backward Model

where $p'_1 = \frac{p_2(1-\alpha)}{\alpha(1-p_1) + (1-\alpha)p_2}$ and $p'_2 = \frac{p_1\alpha}{\alpha p_1 + (1-\alpha)(1-p_1)}$.
 Hence, the capacity of the backward Channel is given by

$$C_b = h\left(\frac{1}{1+2 \frac{h(p'_1) - h(p'_2)}{1-p'_1-p'_2}}\right) - \gamma h(p'_1) - (1-\gamma)h(p'_2)$$

$$\text{where } \gamma = \frac{\frac{1}{1+2} \frac{h(p'_1) - h(p'_2)}{1-p'_1-p'_2} - p'_2}{1-p'_1-p'_2}$$

It is clear that I have given separate parameters for source and channel encoding so that they can be separately decided, and then use equations (1), and then equations (2) to get all the relevant parameters of the code and design.

According to [14], for any rates R_1 and R_2 that $0 < R_1 < R_2 < 1$, there exists an ensemble of LDPC codes with the following property: The ensemble can be punctured from rate R_1 to R_2 resulting in asymptotically good codes for all rates $R_1 \leq R \leq R_2$. Hence, we design punctured codes according to [14] for rate $\min(R_1, R_2)$ and that will work for both and this is the rate we get after the step 2 using the equations (2).

Decoding:

The decoder needs to determine X_1 from information bits of X_1 (partly that were sent from encoder of X_1 and partly sent from encoder of X_2), and the parity bits P_1 and P_3 . Likelihood of these parity bits are decided by the channel noise alone, and similar for the information bits coming from the encoder of X_1 , while the information bits from the encoder of X_2 will have the effect of the BSC in the path from X_1 to X_2 also. Similar procedure will decode the source X_2 .

Remark:

The code construction (irregular LDPC codes) and decoding has to be done in the way suggested in [15] which gives a scheme using density evolution with erasures and errors which is the model of our scheme. So, this model gives the density evolution analysis for this case, and we can choose the degree distribution and decoding parameters according to this density evolution analysis.

III. SIMULATIONS OF JOINT SOURCE CHANNEL CODING FOR ARBITRARY MEMORYLESS CORRELATION

As a first example, take $p_1 = .05$, $p_2 = .185$ and $\alpha = .4$ in Figure 4. This gives a joint entropy of two sources as 1.5. Solving C_f and C_b gives $C_f = 0.4998$, $C_b = 0.4839$. Also take the source rates $R_{X_1} = .8$, $R_{X_2} = .7$, and the channel rates as $R_{c_1} = .9$ and $R_{c_2} = .94$. Putting all these in equations (1) and (2), we get the parameters a, b, c and R as 1.58, 1.4754, 1 and .7259 respectively. These parameters should ideally work for this scheme. To simulate, we use $p_1 = .05$ and $\alpha = .4$ in Figure 4, but we keep p_2 as a variable, changing which we vary the joint entropy of the sources. Also, choose the both the channels as BSC with error probabilities .0129 and .0069 respectively so that both the channels rates are the same as the capacity. The simulation result is shown in Figure 6.

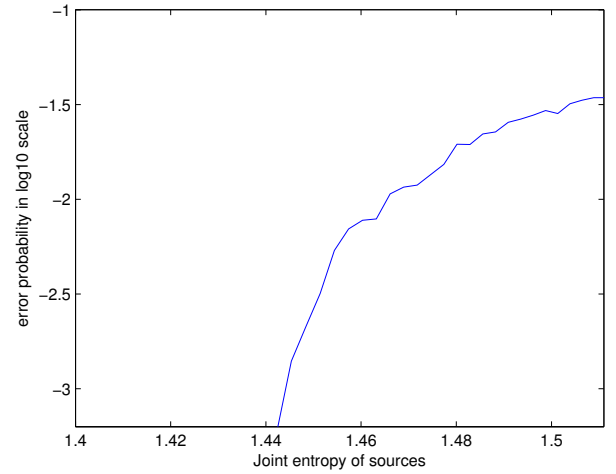


Fig. 6. Simulation Results

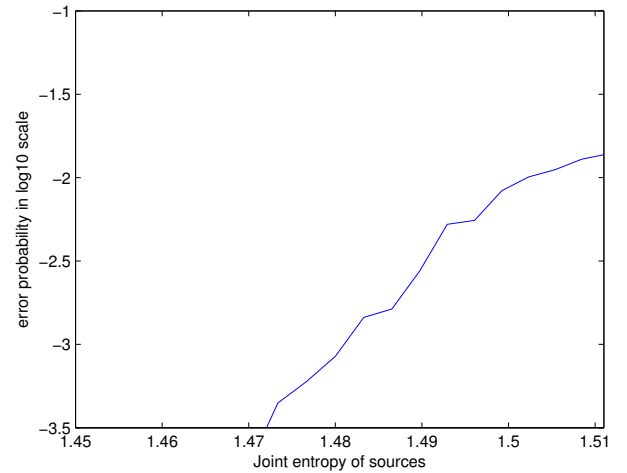


Fig. 7. Simulation Results

As a second example, consider the model for $p_1 = .09$, $p_2 = .143$ and $\alpha = .4$. This gives a joint entropy of two sources

as 1.5. Solving C_f and C_b gives $C_f = 0.4839$, $C_b = 0.4703$. Also take the source rates $R_{X_1} = .8$, $R_{X_2} = .7$, and the channel rates as $R_{c_1} = .85$ and $R_{c_2} = .9$. Putting all these in equations (1) and (2), we get the parameters a, b, c and R as 1.5392, 1.4665, 1 and .7005 respectively. To simulate, we use $p_1 = .09$ and $\alpha = .4$ in Figure 4, but we keep p_2 as a variable, changing which we vary the joint entropy of the sources. Also, choose the both the channels as BSC with error probabilities .0215 and .0129 respectively so that both the channels rates are the same as the capacity. The simulation result is shown in Figure 7.

IV. DISTRIBUTED SOURCE CODING SCHEME FOR MARKOV SOURCE CORRELATION

The correlation between the sources has a memory defined by a Markov model. The two source sequences be X_1 and X_2 as before, with $X_2 = X_1 + N$, where N is generated by a Markov model $\lambda = (A, B, \pi)$, and addition is modulo two addition. The model is characterized by a set of states S_j , $0 \leq j \leq S-1$, the matrix of transition probabilities among states $[A=(a_{ij})]$, with $a_{ij} = P_t(S_j|S_i)$ the probability of transition from state S_i to S_j , and the list giving the bit probability to associate with each state $[B=(b_{jv})]$, with $b_{jv} = P_0(v|S_j)$ the probability of getting output v in state S_j . π is the initial distribution of each state. This model has been taken for the source compression in [16] but this solves the asymmetric case of Slepian-Wolf only. Similar model is also used in [17] for the channel with memory. Some work along the lines of the source correlation having Markov structure for the asymmetric case of Slepian-Wolf problem has also been done in [18]. For the distributed source coding, the results of the previous section still hold with $R_{c_1} = 1$, $R_{c_2} = 1$ and different values of C_f and C_b . Hence, define (R_a, R_b, C_f, C_b) -SW distributed code as the code which returns value of a, b, c and R as the parameters of the code construction as in previous section taking values of R_a, R_b, C_f, C_b by the two step approach in equations (3) and (4)

$$\begin{aligned} (1 - \frac{1}{c})(\frac{1}{R} - 1) &= (1 - C_f)(1 - \frac{1}{a}) \\ (1 - \frac{1}{b})(\frac{1}{R} - 1) &= \frac{1}{a}(1 - C_b) \end{aligned} \quad (3)$$

$$R_a = \left[\frac{1}{a} + \frac{\frac{1}{R} - 1}{b} \right]$$

$$R_b = \left[1 - \frac{1}{a} + \frac{\frac{1}{R} - 1}{c} \right]$$

$$R' \leftarrow \frac{R}{R + \max(\frac{1-R}{b}, \frac{1-R}{c})}$$

$$b \leftarrow b \times \frac{R(1-R')}{R'(1-R)} \quad (4)$$

$$c \leftarrow c \times \frac{R(1-R')}{R'(1-R)}$$

$$R \leftarrow R'$$

Let us describe the encoding and decoding process in detail
Encoding:

We have two sources X_1 and X_2 each of k bits. The scheme has been shown in Figure 8. Send a portion $k \cdot \alpha$ of the bits

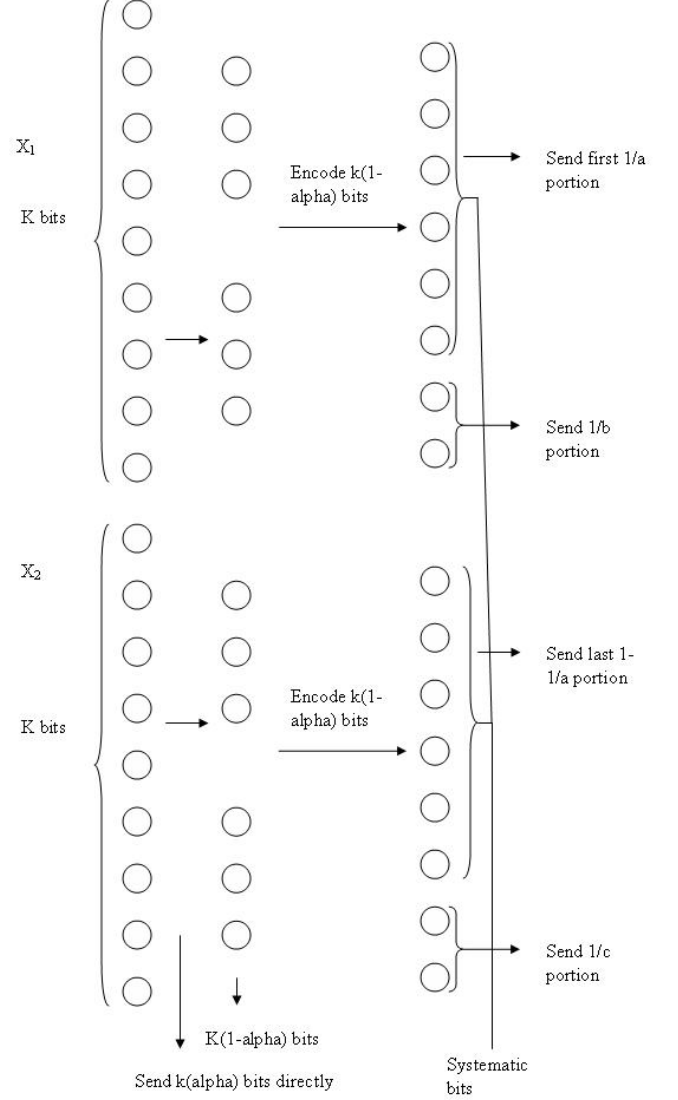


Fig. 8. Encoding scheme for Markov correlation model

equally spaced directly to the receiver. From the remaining $k(1-\alpha)$ bits, encode them using $((R_{X_1} - \alpha)/(1-\alpha), (R_{X_2} - \alpha)/(1-\alpha), C_f, C_b)$ -SW Distributed code defined earlier where R_{X_1} , R_{X_2} , C_f and C_b are respectively the desired source rates for the two sources and the forward and backward capacities of the channel between the two sources.

Decoding:

The decoding approach has been shown in Figure 9. The steps of message parsings in the scheme are:

- 1) Message Parsing from the bit nodes to the Markov Model: The systematic bits of the two sources and the $k \cdot \alpha$ bits that were sent without any encoding are used to get the likelihoods of the bits of N , as N is the sum of the two sources, and hence we can easily get their likelihoods from the likelihoods of the bits of the two sources. The likelihoods of the nodes of the received vector corresponding to the systematic bits of the two sources be a_n^x and b_n^x respectively with n denoting the n^{th} bit with n varying from 1 to k and the ordering is

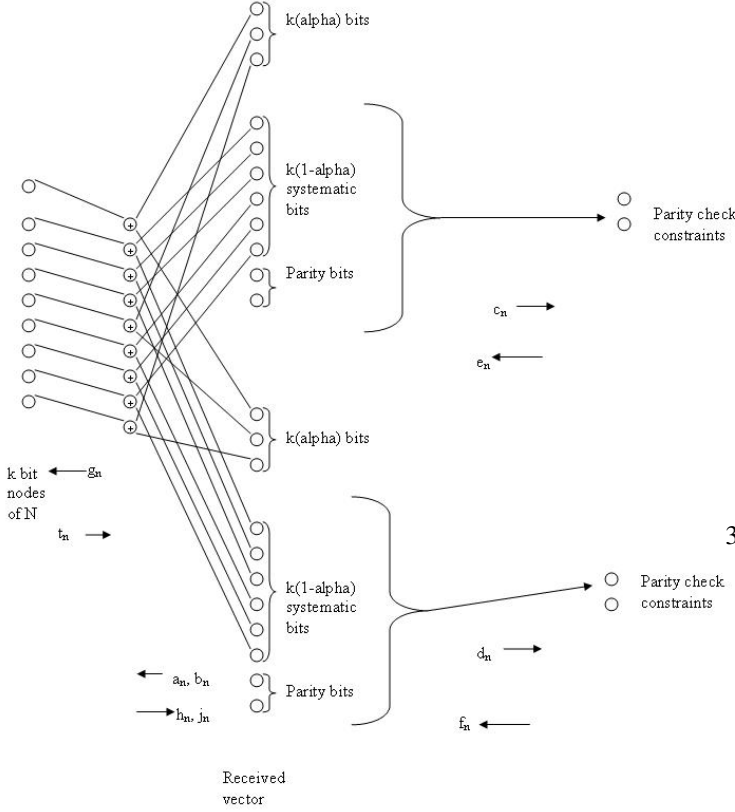


Fig. 9. Decoding scheme for Markov correlation model

according to the ordering in the input source vector of the two sources, then the likelihoods of the nodes of N g_n^x are given by

$$g_n^0 = a_n^0 \times b_n^0 + a_n^1 \times b_n^1$$

$$g_n^1 = a_n^0 \times b_n^1 + a_n^1 \times b_n^0$$

- 2) Message Parsing from Markov Model to the bit nodes: After receiving the noisy version of N , and knowing the memory structure of N , we can use trellis decoding to update the likelihoods of N and send the likelihoods towards the bit nodes. Consider the trellis for a finite-state binary Markov model. The starting and ending states associated with a particular edge ' e ' are represented by $s^S(e)$ and $s^E(e)$, respectively, and the bit corresponding to ' e ' is denoted by $\epsilon(e)$. The trellis has two parallel branches between states (one associated with the bit $\epsilon(e) = 0$ and the other with the bit $\epsilon(e) = 1$). Each one of the branches in the trellis will have an associated a priori probability a_e , which is obtained from the parameters of the Markov Model. The resulting equations that implement the belief propagation algorithm over the Markov model are given by

$$\alpha_k(s) = \sum_{e: s^E(e)=s} \alpha_{k-1}[s^S(e)] a_e g_k^{\epsilon(e)}$$

$$\beta_k(s) = \sum_{e: s^S(e)=s} \beta_{k+1}[s^E(e)] a_e g_{k+1}^{\epsilon(e)}$$

$$t_n^x = \eta_n \sum_{e: \epsilon(e)=x} \alpha_{n-1}[s^S(e)] \beta_n[s^E(e)] a_e g_n^x$$

where η_n is the normalizing factor so that $t_n^0 + t_n^1 = 1$. Now, this message t_n^x goes back to the systematic portion of the bit nodes. Using t_n^x we update the less reliable of the systematic bit nodes of the two sources by the xor operation. The systematic bits of two sources are updated by messages h_n^x and j_n^x respectively. Let us consider message update of h_n^x , j_n^x will be similar. The update equations for h_n^x are given by

$$\begin{aligned} |.5 - a_n^x| > |.5 - b_n^x| &\Rightarrow h_n^0 = a_n^0 \times t_n^0 + a_n^1 \times t_n^1, \\ b_n^1 &= a_n^0 \times t_n^1 + a_n^1 \times t_n^0 \\ |.5 - a_n^x| \leq |.5 - b_n^x| &\Rightarrow h_n^x = b_n^x \end{aligned}$$

Since one of a_n or b_n will be known perfectly (by the encoding scheme), this just means to update the other if that is not known perfectly.

- 3) Message Parsing from the bit nodes to the check nodes: This message propagation is from the $k(1-\alpha)$ systematic bits and the parity bits to the check nodes using their respective likelihoods, likelihoods of the systematic nodes given by h_n^x and j_n^x and those of parity bits the same as the initial likelihoods in the first iteration and those updated from the previous iteration from check nodes to bit nodes later. This message parsing is the same as the standard belief propagation scheme
- 4) Message Parsing from the check nodes to the bit nodes: Using the belief propagation, we send the updated likelihoods towards the bit nodes.

Hence, the overall scheme can be summarized as:

- 1) Message parsing from bit nodes to Markov model and back to the bit nodes.
- 2) Message parsing from bit nodes to check nodes.
- 3) If the parity equations are not satisfied, and iterations are less than maximum-iterations (say 100) go to Step 1 after message parsing from the check nodes to the bit nodes.

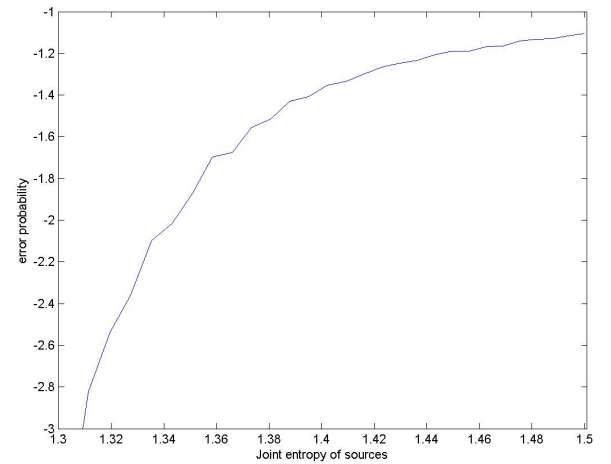


Fig. 10. Simulation Results

V. SIMULATIONS OF DISTRIBUTED SOURCE CODING FOR MARKOV CORRELATION

Take the source correlation model as a Markov source with two states S_1 with probability of 0 as one, and S_2 with probability of 0 as zero, and the transition probability from S_1 to S_2 and from S_2 and S_1 are the same and equal to p , the parameter which decides the entropy rate in the plot. Also, take the first source to be i.i.d, making $C_f = C_b = 1-h(p)$. Take $k=6250$, $\alpha=2$, and $R_{X_1} = R_{X_2} = 0.75$. Hence, we send $6250 \cdot 2 = 12500$ bits of each source without encoding and for the remaining bits we use the equations (3) and (4) with $R_a = R_b = .6875$, and $C_f = C_b = .5$, we get $a=2$, $b=1$, $c=1$, $R=.8421$. The simulation with varying value of p is shown in Figure 10.

VI. CONCLUSIONS

This work deals with the duality between source and channel coding. We take ideas developed for channel coding and transform them appropriately to construct joint source channel coding techniques. Distributed source coding scheme is a special case of the above work with the capacity of the channels to be equal to 1. We illustrated how to do joint distributed source channel coding at arbitrary source rates in the Slepian-Wolf rate region with arbitrary memoryless source correlation and arbitrary channel. We also illustrated how to do distributed source coding at arbitrary point in the Slepian-Wolf rate region when the source correlation model has memory. In all the simulations, we considered regular $(3,x)$ LDPC codes. Taking irregular codes instead may perform better due to irregularity in the coding scheme.

Further work can be done to extend the distributed source coding for sources with Markov correlation to joint source channel coding. Joint source channel coding for sources with memory has been studied in [27][28]. Some work on the asymmetric case for this problem has also been done using turbo codes in [29]. But the problem using LDPC codes and doing for general source rates are still open problems.

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